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Complex modeling of complicated processes: analytical model parameters adaptation in solving qualimetric problems

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Abstract. The relevance of the study is due to the fact that despite the sustainable development of methods and technologies for creating models of various classes and purposes, the tasks of managing their quality (the so-called inverse problems of theoretical qualimetry) are often neglected. For example, in the field of information technology, much more attention is paid to assessing the quality of software products, which is reflected in the corresponding standards. These tasks become especially relevant when models reflect a complex process (CP), and such processes may be unique, and obtaining complete objective data on such CPs prior to modeling is difficult or impossible (accidents and disasters models, military operations, natural disasters, and others like that). The article's main purpose is to consider the issues of solving the inverse qualimetric problem by parametric adaptation of analytical models to unpredictable possible changes in the complex processes' nature. The solution to this problem is based on the combined (analytical and simulation) modeling of unique processes. Theoretical qualimetry involves a quantitative assessment of any product's qualitative properties; to ensure the adequacy level of analytical models (based on the results of their parametric adaptation), it is planned to evaluate with the use of well-known metrology principles regarding relative measurement deviations. In this case, detailed simulation modelling results are used as a reference for combined modelling. The complexity of the processes conditiond by the expediency of their polymodel description in order to mutually compensate for the objective deficiencies of heterogeneous models, as well as the use of simulation capabilities to adjust the parameters of analytical models in cases where the use of the latter is due to strict time constraints on obtaining calculation results and developing control actions based on them. A variant of the formalized statement of the problem of parametric adaptation of analytical models is presented. A presented example that considers the adaptation of parameters of the Lanchester-type analytical model conducted on simulation data. The adaptation of parameters could be carried out in different ways, and the expediency of choosing a certain one is determined by the assessment of mutual deviations of the results of heterogeneous modeling. The results of the calculations witnesses that parametric adaptation can be an effective element in solving one of the key tasks of model qualimetry, namely, ensuring their required qualitative properties. The prospect of further research is to consider the issues of structural adaptation of simulation models to the given conditions for displaying complex processes.

Keywords: simulation and analytical modeling; model qualimetry; Lanchester-type models; parametric model adaptation.

Introduction

In the modern world, a large number of methods and techniques are used for quantifying and assessing the various types of products' quality. The science that studies and combines these methods and techniques, systematizes and develops them is called qualimetrics (from the Latin word qualitas – quality, property and the ancient Greek metreon – to measure). Qualimetrics is a scientific discipline within the framework of which the methodology and problematic issues of complex, quantitative assessment of object quality are studied (Kuts, Stoliarchuk, Druziuk, 2012), (Tsyba, 1997), (Bozhenko, 2003). E. Hevorkian (2022) gave the following definition: qualimetry is a scientific field that studies and develops principles and methods of quantitative quality assessment; establishes quantitative regularities formation of various product quality indicators in design projecting, manufacturing and exploitation fields; discloses objective links between the costs of creating products or articles and the beneficial effect achieved as a result of these costs. The term "qualimetry" concisely and accurately conveys the meaning of the concept of "quality measurement" (Shapoval, 2002). In qualimetrics, two main concepts are used - measurement and evaluation. Measurement is understood as finding the value of a physical quantity by means of research methods with the help of special technical means. Evaluation in qualimetry is used both differential (single) and complex, depending on the need.

Theoretical qualimetry involves for the distinction between direct and inverse tasks, which are understood as, respectively, the tasks of direct assessment of quality properties (direct) and the tasks of quality management in order to ensure the required properties of products for any purpose (inverse). The structure of a comprehensive simulation model quality assessment is presented in the article by B. Butvin, M. Mashkin, O. Sobolev etc. (2022). Since the models themselves are created both for the purpose of analyzing existing original objects and for the purpose of their synthesis, the inverse tasks play a major role in model qualimetry. According to the fair statement of the authors B. Sokolov, R. Yusupov (2004), B. Sokolov, R. Yusupov, S. Mikoni (2014, 2018), despite the sustainable methods development and technologies for creating models of various classes and purposes, such tasks often remain beyond attention. These tasks become especially relevant when the models reflect a complex (unique) process (CP), and obtaining complete objective data on such CP's prior to modeling is difficult or impossible. A characteristic manifestation of the process complexity from the cybernetic point of view, according to S. Mikoni, B. Sokolov, R. Yusupov (2018), M. Moiseiev (1981), is considered to be the necessity of its polymodel description, involvement of heterogeneous models based on different methodological frameworks. The use of heterogeneous models combined into a complex will not only increase substantiation of assumptions about the probable nature of the complex process's course, but also provide additional opportunities for solving qualimetry model inverse problems. Among the two main types of modeling – analytical and simulation – simulation models (SM) have much broader capabilities to reflect key factors of complex random processes. At the same time, despite the rapid development of distributed computing technologies, SM remains the main processor time consumers. Analytical models (AM) provide much higher efficiency in calculations, but conceding to SM in terms of adequacy of the CP display. In conditions when the calculations efficiency is a priority requirement for modeling, parametric adaptation of AM (adjustment, parameters correction to the variable conditions of CP simulating) is an effective way to increase the models' adequacy while maintaining the calculations' efficiency. Solving the inverse qualimetric tasks in this way is a relevant practical task, which led to the article's topic choice.

Materials and Methods

To avoid terminological divergences in the quality of model adequacy definition, the following is accepted (Moiseiev, 1981), (Ahrem, 2007), (Rumiantsev, 2010): it is the ability of the model to take into account the most significant factors that affect the course of the CP. The SM essence implies the rationality of reflecting all stages of the modeled process while maintaining the logical sequence of their course in time, and taking into account random, difficult to formalise factors using pseudo-random variables with appropriate laws of their distribution. Along with the possibility of using almost the entire arsenal of analytical modeling methods in SM, this ensures CP' sufficient description detailization and potentially high adequacy of such models (of course, they must pass all verification stages). In the absence of real data on the course of the CP, the simulation modelling results can serve as a reference, a benchmark in relation to the AM. In this case, with the comprehensive use of AM and SM, a possible variant of the AM parametric adaptation task is presented as follows.

The article considers a poly-model complex (PMC) designed to predict probable variants of the CP's course and developing controlling influences (CI) based on the calculations results. The PMC includes a reference simulation model (IM_e) and analytical models' plurals $\{AM_a\}$, $a = \overline{1,A}$ with a simplified process description, which provide the necessary efficiency of modeling and obtaining the entire set of calculated indicators $(\bar{Y})$ required for the CI production. If it is necessary to develop the CI as a response to changes in the CP' course, time restrictions are imposed on the time of obtaining the $(\bar{Y})$ indicators. Such limitations determine the possibility of using IM_e only at the stage of preliminary forecast calculations of possible options for the CP development, with its beginning and, if necessary, in response to changes in its course, the capabilities of the complex set $\{AM_a\}$ are used exclusively. In order to ensure the AM' adaptation to changes in the conditions of CP modeling, it is necessary:

1) to determine, with regard to the entire set of l parameters $\bar{P}_l^{(a)}$ of each a-th analytical model of the PMC, the appropriate order of their coordination with similar parameters obtained by simulation modeling $(\bar{P}_l^{(e)})$. The use of agreed parameters in AM should minimize the current deviation (absolute or relative, ΔY) between the results of analytical and simulation modeling, while such deviation cannot exceed some established limit value (ΔY_D):

$$\begin{gathered}\Delta Y_{min}\left[AM_a\left(\bar{P}_l^{(a)}\right); IM_e\left(\bar{P}_l^{(e)}\right)\right] \le \Delta Y_D; \\ \overline{P_h}^{(a,e)} = \left\{P_1^{(a)}, \dots, P_l^{(a)};\ P_1^{(e)}, \dots, P_l^{(e)}\right\},\ \ l = \overline{1,L}.\end{gathered} \tag{1}$$

2) to numerically assess the possibility of ensuring the AM adequacy to the changes in simulated CP conditions by matching the analytical models' parameters with the reference SM parameters.

Based on the results of the required runs number IM_e, the parameters $(\bar{P}_l^{(e)})$ will be obtained in the statistical data form. The adjusted parameters $(\bar{P}_l^{(a)})$ can be presented in the form of mathematical expectations' estimates (based on the statistical data processing results) and as variable values. Depending on the analytical models' class included in the PMC, the use of the obtained distributions makes it possible to use them to refine the AM parameters in various ways. To illustrate, let's assume that the indicators' analytical calculation $(\bar{Y})$ is carried out by means of differential equation systems. In this case, the totality of parameters $(\bar{P}_l^{(a)})$ the coefficients on the variables and the free terms of the equations will be acting. An explanation regarding the refintment order for clarifying the AM parameters can be demonstrated in the following example.

Let that with help of the PMC, the course and results of combat actions are been modelled, and one of the necessary totality indicators $(\bar{Y})$ is the number of opposing groupings at certain moments

of time, which are analytically determined by modifying analytical Lanchester models (Fursenko and Chernovol, 2020). Let's also assume that, under tasks conditions, such an AM should take into account the recovery of parties' numbers (instead of the lost ones) and has the form the following equations system:

$$\begin{cases} \frac{dx}{dt} = (-c - b)y + d[x(0) - x]; \\ \frac{dy}{dt} = (-g - f)x + h[y(0) - y], \end{cases} \quad (2)$$

where x, y are the numbers of opposing groupings at time t ≥ 0;

x(0), y(0) – initial conditions of the groupings' number at the action starting moment (t = 0);

c, g, b, f – intensity of the parties' losses due to the impact of different types of armaments;

d, h – the intensity of the parties' numbers recovery.

Despite the uniqueness of each combat plans implementation, the general trends in such a joint venture development, proceeding, in particular, on previous experience in similar conditions, are approximately known. That is, some averaged values of the AM parameters are also known, assuming, for example, that the parameters *c*, *g* and *b*, *f* characterise losses from the effects of planar and point means of destruction, respectively. A variant of such parameters can take the following values (Table 1):

Table 1. Variant of the averaged (based on the AM results) parameters' values $\overline{P_l}^{(a)}$ of the equations system (2)

x side	***c***	***b***	***d***
	0,0087	0,0121	0,0179
y side	***g***	***f***	***h***
	0,0107	0,0058	0,0264

For combat actions' particularised conditions and for the purpose of AM type parametric adaptation (2), based on simulation modelling results obtained the mentioned parameters distributions and the CP sought indicator values (in this case, the number of opposing parties x_{IM_e} and y_{IM_e} at a combat actions' certain moment of time). In accordance with the assigned task, it is necessary to minimize the discrepancy between the CP indicators obtained by analytical and simulation way, with simultaneous compliance with the condition $\Delta Y \leq \Delta Y_D$. It can be concidered that for the type of model under consideration, the permissible deviation limits are about 15 % ($\Delta Y_D \approx$ 15 %). The deviation, i.e., ΔY, could be estimated using a well-known expression to determine the calculations' relative error (regarding the defined indicators):

$$\delta_x^* = \frac{|x_{IM_e} - x_{AM}^*|}{x_{IM_e}} \cdot 100; \quad \delta_y^* = \frac{|y_{IM_e} - y_{AM}^*|}{y_{IM_e}} \cdot 100, \quad (3)$$

where x_{AM}^*, y_{AM}^* - the numbers of parties' opposing groupsings at a certain moment of time, established by the results of analytical modeling with adjusted parameters; deviations for AM with averaged parameters are determined in the same way (denoted by $\delta_{x,y}$, respectively).

In fact, expression (3) can serve to numerically evaluate the level of analytical modeling adequacy to simulation in the CP complex modeling.

The choice of parameters' form that can be used as corrected parameters in the ($\overline{P_l}^{(a)}$), is is determined by the look of the obtained statistical distributions and the results of the relative deviations assessment $\delta_{x,y}$. In view of the above, mathematical expectation estimates (MEE) of the obtained

data ($\tilde{m}[\, x_{IM_e};\, y_{IM_e}]$), are used as refined parameters, i.e. model (2) will appear as a system of linear non-homogeneous equations. Let us assume that, based on the results of simulating the CP specific conditions, the following assessments MEE of the system parameters' statistical data (2) were obtained, Table 2:

x side	***c***	***b***	***D***
	0,0141	0,0152	0,0302
y side	***g***	***f***	***H***
	0,0137	0,0078	0,0264

For certainty, let us assume that the analysis of actions is carried out on a time interval up to t = 5 (days) with arbitrarily taken initial conditions x(0) = 30000; y(0) = 25000. The indicators' values obtained by IM_e, are: $\tilde{m}[\, x_{IM_e}(5)] = 23173$; $\tilde{m}[y_{IM_e}(5)] = 19051$. Comparative estimates of the system numerical integration results (2) for the certain initial conditions are shown in Fig. 1 It is also clear that changing (adjustment) the parameters of such an AM type determines the necessity of assessing the stability (in the Liapunov sense) of the decicions obtained. For the parameters of Table 2 and the type system (2), the determinant is represented as:

$$\begin{bmatrix} -0{,}0302 - k & -0{,}0293 \\ -0{,}0215 & -0{,}0264 - k \end{bmatrix}, \tag{4}$$

respectively, the characteristic equation:

$$k^2 + 0{,}0566k + 0{,}000167 = 0, \tag{5}$$

roots: k_1=-0,0031,k_2=-0,0535.

The roots type meets the conditions for the stable decicions existence.

The established constraint assessment carried out in accordance with (2), let us affirm that the use of the refined parameters ensures a reduction in the deviation of the simulation and analytical modeling results to the permissible difference, from δ_x=19,23 % and δ_y=19,56 % to δ_x^*=15,68 % to δ_y^*=16,27 %, see Fig. 1.

The CP physical essence and the simulation results indicate the nececity to take into account the influence of a random factors' plurals, the reflection of some CP factors is possible in the AM form as stochastic differential equations (SDE) systems (Debrabant, Roessler, 2008), (Clauden, 1992). Thus, for the considered conditional variant, the parameters' statistical series (in the form of histograms) are presented in Fig. 2. An example of a simulation, given, let us suppose, for the parameters *b* and *d*, is obtained from the results of 40 independent runs IM_e and grouped into 10 digits of their values (I_i).

Histograms view of the statistical series indicates that the obtained parameter b values are grouped around the statistical average, while the parameter d values have a pronounced character of small fluctuations over the entire interval of their possible values. Under the condition that similar character will be seen in other equivalent parameters of system (2), such a model can be represented in the form of a stochastic differential equations (SDEs) system (Byram, Partal, and Buyukoz, 2018), (Allen, 2007), (Wang, 2016) with multiplicative Wiener noise. In this case, taking into account the Table 2 data, the system of SDEs is presented as follows:

$$\begin{cases} \frac{dx}{dt} = -0{,}0293y + 0{,}0302[x(0) - x]dw; \\ \frac{dy}{dt} = -0{,}0215x + 0{,}0264[y(0) - y]dw, \end{cases} \quad (6)$$

dw – random Wiener process implemented as: $dw = \varepsilon\sqrt{dt}$, $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n), \varepsilon_i \sim N(0,1)$ – normal distribution with zero average and unit variance.

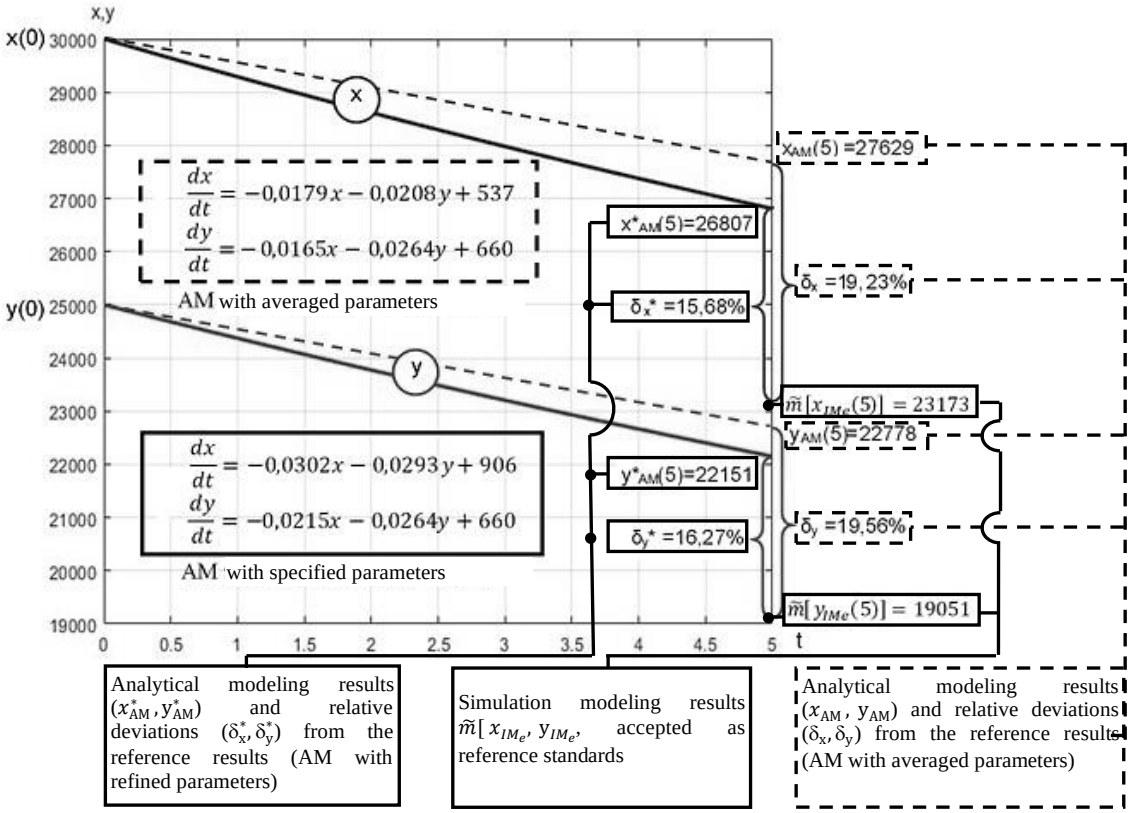


Figure 1. Simulation results and analytical modeling comparison; system solution (2) for the initial conditions with the parameters of Tables 1 and 2

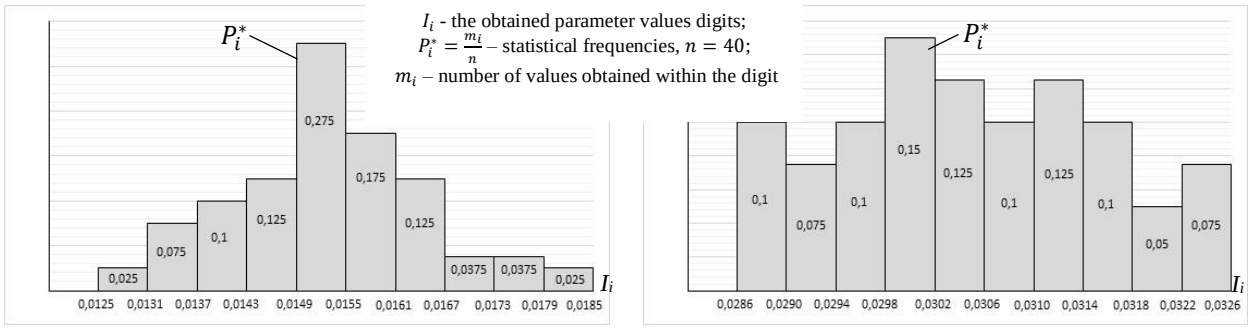


Figure 2. Histograms of system parameters statistical series (2):
a) parameter *b*; b) parameter *d*

The use for the purpose of solving system (6) of specialised methods of PDE numerical integration (Lukshin and Smirnov, 1990), (Averina and Artemev, 2005), (Tocino and Ardanui, 2002) allows to obtain results in the form of both approximate strong and approximate weak decisions.

Weak decisions are used to find the probability distributions of the desired values, at the same time, individual realisations are not crucial. In the context of the AM parameter adaptation task (based on simulation calculations), the obtained probability distributions for weak solutions are of greater interest. The results of numerical solution system (6) by the Milstein method (in the weak sense) are presented in Fig. 3.

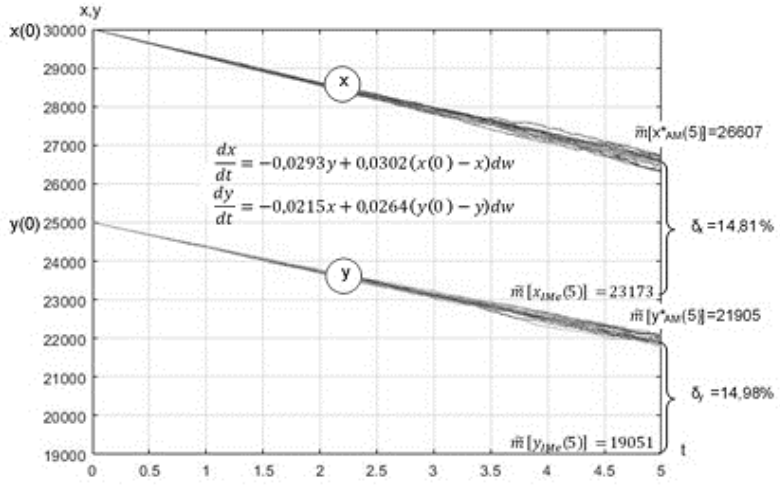


Figure 3. The system solution's results (2) for the defined initial conditions by the Milstein method (fragment for 20 trajectories)

Assessing the solution's stability, as in the ordinary differential equations' case, is an important model's qualitative characteristic (6), but in the case of using SDE, the stability concept has a somewhat broader interpretation, in particular it is related to the methods of solving the such equations (Lukshyn and Smyrnov, 1990), (Averina and Artemev, 2005), (Carletti and Berraj, 2004). Taking into account the statements by Lukshyn and Smyrnov (1990), Mao (2007), and Averyna and Artemiev (2005), as well as the confirmed stability of the deterministic basic system (Fig. 2), the results of solving (6) are considered stable. For their type shown in Fig. 3, the relative deviations of the selected indicators (δ_x та δ_y), calculated from the MEE distributions for 10,000 system realizations (6) are δ_x = 14,81 %; δ_y = 14,98 %. In other words, the condition $\Delta Y \leq \Delta Y_D$ is met in this case, which indicates that the AM is satisfactory level of adequancy.

Result

The given simplistic example demonstrates the necessity for a reasonable choice of the refined parameters application form, as a result of the all the data analysis of the CP simulation modeling. Similarly, the parameters' form is refined in the case of a significantly non-stationary process (O. Mashkin, 2020), i.e., an of AM form example (2) will reflect the process as a system of linear non-stationary equations (or combined, as a system of non-stationary SDE). If the reasonable use of the refined parameters established forms does not ensure the fulfillment of the condition $\Delta Y \leq \Delta Y_D$, this will indicate the necessity to adjust the type or class of the AM itself, not just its parameters. So, only when the detailed simulation and analytical modelling results of unique CP achieve convergence (within certain bounds), we can talk about the possibility of elaborating a CI based on the analytical calculation results.

In addition, practical calculations have confirmed that increasing the number of simulation runs (with further AM parameters refinement in the manner considered) also contributes to ensuring the consistency of the heterogeneous modeling results, and, accordingly, to increasing the AM adequacy.

Discussions

Models' parametric and structural adaptation are covered in thematic publications (in particular, S. Mikoni, B. Sokolov, R. Yusupov (2018), B. Sokolov, R. Yusupov (2004), M. Moiseiev (1981)) as appropriate methods for ensuring the necessary models' qualitative properties. However, the recommendations in these sources are mostly a general theoretical nature and are limited to models of real-world objects-originals. Besides, the absence of a well-established terminology base in the field under consideration adds certain difficulties in solving this problem. Discrepancies in the understanding of the concept of model adequacy, different opinions on the principal possibility and methods of quantifying the adequacy of SP unique models do not contribute to the unambiguous establishment of such a qualitative characteristic from the qualimetric point of view. According to the right remark of the authors S. Mikoni, B. Sokolov, R. Yusupov (2018), summarising the results of applied research in the area under consideration should become the basis for further development of this promising scientific and practical direction.

Conclusion

The solution of the inverse qualimetric problem in the combined display of unique SPs through AM parametric adaptation will require, first of all, the formation of a PMC as part of coordinated CP analytical and simulation models, in which at least one (or a single) simulation model should serve to obtain reference results. Thus, the appropriate course of action regarding adapting the AM parameters is presented in the following sequence:

- Identification of the analytical models' most important parameters are in need to be refined by simulation;

- determining the distributions of parameters by runs of reference AM for the most predictable variants of the CP' course;

- use of the parameters refined by simulation in analytical models in the form determined by the simulation results analysis (the form of the statistical data set), the class of analytical models used and the peculiarities of their software implementation;

- assessing the mutual deviation of analytical and simulation modelling results, adjusting (combining), if necessary, the forms of the refined parameters applied. In the case inability to comply with the requirement for consistency of heterogeneous modelling results, the type of analytical models used will be needed to be clarified.

That is, analytical models' parametric adaptation can be considered an effective element of solving one of the key tasks of model qualimetry, that is, ensuring their necessary qualitative properties in the complex application of complex processes heterogeneous models.

The further research prospect is to consider the questions of SM structural adaptation to determine the necessary data for the parametric adaptation of AM.

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