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An approach to evaluating opposing parties over time based on the analytical solution of a system of nonlinear differential equations using the method of average dynamics

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Abstract The relevance of this research stems from the need for a critical analysis of a rapidly developing military conflict (as an integral part of military security). It can be described by the nonlinear Lanchester equations. This critical analysis focuses on two aspects. The first aspect allows to establish a formula-driven relationship of the initial force ratio with predetermined criterion values. This relationship takes the form of a biquadratic equation. The second aspect concerns the critical assessment of the time required to achieve the ultimate objective of one of the parties in a conflict, where the original system is reduced to a single nonlinear first-order differential equation that can be solved for the time parameter using tabular integration. The aim of the article is to develop an approach for calculating the initial ratio of firepower available to the warring parties, as well as the predicted duration of hostilities that would produce an acceptable extent of losses for one of the parties while achieving the final predefined firepower ratio. Conventional approaches based on the dynamic averaging method, allow to calculate the required initial ratios of forces and assets of the warring parties using models describing conflicts of a certain intensity based on linear differential equations with a constant destructive effect under predefined (acceptable) loss thresholds. At the same time, the use of mathematical models based on systems of nonlinear differential equations to describe combat operations allows to account for the intensity of the destructive effect, which, as is common for modern military conflicts, changes over time. Nowadays, systems of these equations are usually solved using numerical methods, which is a rather laborious task. The article presents an approach based on the analytical solution of a system of nonlinear differential equations using the dynamic averaging method, which takes into account the intensity of the destructive effect, which changes in the course of hostilities. Such an approach makes it possible to quickly calculate the parameters that can be used in operation (combat) planning (in particular, when justifying the required force composition and duration of an operation). The suggested method allows to calculate the initial ratio of firepower available to the warring parties, as well as the predicted duration of hostilities that would produce an acceptable extent of losses for one of the parties while achieving the final predefined firepower ratio. The feasibility of this approach is confirmed by a computational example.

Keywords: analytical analysis, mathematical model, dynamics, combat operations, conflict, correlation.

Introduction

Applied research in the field of conflict between warring parties is quite multifaceted and concerns a wide range of interdependent processes that require extensive calculations. The analytical calculations are based on methods of mathematical analysis (differential and integral calculus) with the subsequent transition into the field of optimal control theory (Kotliarov, 2020), primarily in the part that concerns the choice made by one of the warring parties regarding the best (necessary) process parameters or their optimal ratios, for example, the force ration throughout the entire duration of the conflict. When assessing this parameter, the most important task is to determine the appropriate initial ratio of firepower available to the warring parties, as well as the predicted duration of hostilities that would produce an acceptable extent of losses for one of the parties while achieving the final predefined firepower ratio. The article is presented in a manner that provides a quantitative assessment of all these variables in relation to conflicts of varying intensity (the least studied in the specialized literature).

It is worth noting that the existing methods for calculating the force ratio are mainly based on assessing the combat capabilities of the warring parties, which are calculated by summing up their combat potentials (Leontiev, 2009), (Zahorka, 2020). Then the assumed initial force ratio of the warring parties is determined by taking into account the value of the combat sustainability factor.

A similar approach was considered by Leontiev (2009), and Mozharoskyi (2019) when describing the method of static modeling used to determine the combat potential of troops (forces). However, as for the value of the combat sustainability coefficient, it is usually determined based on the lessons learned from wars and military conflicts and it can have a fairly wide variation margin. Moreover, given static problem is somewhat incomplete in terms of the time-based analytical interrelation between the required initial force ratio and the final required values of the relevant indicators at the end of hostilities.

The dynamic averaging method can help address this issue. By applying this method to models describing conflicts of varying intensity based on linear differential equations with a constant destructive effect, researchers obtained the corresponding mathematical dependences (Venttsel, 1964), (Zahorko, 2020), (Romanchenko, 2020).

At the same time, analysis of lessons learned from deterring Russian armed aggression against Ukraine (Tom Cooper, 2022), (Zaluzhnyi, 2023), shows that the intensity of the destructive effect exerted by both parties can change in the conditions of prolonged hostilities if one or both parties lack forces and assets for a rapid offensive (offensive operation along Kherson axis) or rapidly evolving hostilities (offensive operation along Kharkiv axis). Mathematical models based on systems of nonlinear differential equations are better suited to take into account the rapidly changing combat environment. At present, systems of such equations are usually solved using numerical methods, which is a rather laborious task that requires a long time, special equipment, software, and large amounts of input data.

The aim of the article is to develop an approach for calculating the initial ratio of firepower available to the warring parties, as well as the predicted duration of hostilities that would produce an acceptable extent of losses for one of the parties while achieving the final predefined firepower ratio.

Methodological Framework

First of all, let's formalize the problem to be solved.

Let's say there are two warring parties - A and B.

The following values are given: for A party – the initial amount of firepower $A(0)$ and the average intensity of the actual destructive effect α; for B party – the average intensity of the actual

destructive effect β.

Moreover, the following restrictions are set at the start $t = T$ of the operation (onset of hostilities):

$$\begin{cases} K(t) = K(T)_{необх}; \\ R(t) = R(T)_{доп}, \end{cases} \quad (1)$$

where $K(T)_{необх}$ is the final firepower ratio to be achieved by the parties;

$R(T)_{доп}$ – the acceptable level of firepower losses for the *B* party.

To solve the problem, it is necessary to determine the required initial ratio of firepower of the warring parties $K(0)_{необх}$ the projected duration of the operation *T*, which satisfies the conditions (1).

At the same time, the dynamic changes in the number of firepower assets available to the warring parties can be described by a system of nonlinear differential equations that takes into account the intensity of the destructive effect, which changes in the course of hostilities. According to (Venttsel, 1964), (Mashkin. 2019), and (Romanchenko, 2022) it is recorded as:

$$\begin{cases} \frac{dA(t)}{dt} = -f_B(t)B(t); \\ \frac{dB(t)}{dt} = -f_A(t)A(t), \end{cases} \quad (2)$$

where $A(t), B(t)$ is the number of firepower assets of the forces of the warring parties;

$\frac{dA(t)}{dt}, \frac{dB(t)}{dt}$ – the dynamics of changes in the number of firepower assets available to the warring parties over time;

$f_A(t), f_B(t)$ is the intensity of the destructive effect of the firepower assets available to the warring parties (non-linear values). According to (Romanchenko, 2022) $f_A(t) = \alpha \frac{A(t)}{B(t)}$, $f_B(t) = \beta \frac{B(t)}{A(t)}$;

t is the current duration of the operation (hostilities).

The proposed approach envisages two stages of calculations.

Stage one involves calculating the required initial ratio of firepower $K(0)_{необх}$.

Let's introduce some metrics that are typically used to assess warring parties into this problem. These metrics include, first of all, the ratio between the number of firepower assets available to the warring parties:

$$K(t) = \frac{A(t)}{B(t)},$$

which is important to assess both at the initial stage of the operation (hostilities) and at the final stage at the given point in time $t = T$. Let's document these relationships as follows:

$$K(0)=\frac{A(0)}{B(0)};\qquad K(T)=\frac{A(T)}{B(T)}, \tag{3}$$

where $K(0)$ is the initial ratio of firepower assets of the warring parties at *t=0;*

$K(T)$ is the ratio of firepower assets of the warring parties at the time $t=T$.

Let's introduce a metric:

$$R(T)=\frac{B(0)-B(T)}{B(0)}, \tag{4}$$

that characterizes the relative losses of party *B* at a given time $t=T$.

Dividing the first equation of the system (2) by the second, the result is as follows:

$$\frac{dA(t)}{dB(t)}=\frac{\beta}{\alpha}\frac{B^3(t)}{A^3(t)}. \tag{5}$$

From there, integration gives us:

$$A^4(t)=\frac{\beta}{\alpha}B^4(t)+C, \tag{6}$$

where *C* is an arbitrary integrating constant.

Formula (6) is true for any given moment in time and thus, in general, establishes an analytical relationship between the number of firepower assets of the warring parties. In that case, for the beginning of the operation (hostilities), this formula (6) can be written as follows:

$$A^4(0)=\frac{\beta}{\alpha}B^4(0)+C, \tag{7}$$

and for the $t=T$ point in time:

$$A^4(T)=\frac{\beta}{\alpha}B^4(T)+C. \tag{8}$$

Eliminating from the functions (7), (8) the integration constant *C* and taking into account the relations (3), (4), one gets the general function for $K(0)$:

$$K(0)=\sqrt[4]{K^4(T)(1-R(T))^4+\frac{\beta}{\alpha}(1-(1-R(T))^4)}. \tag{9}$$

By replacing in (9) the parameters $K(T)$ and $R(T)$ with their required values $K(T)_{необх}$ and $R(T)_{доп}$, one gets a formula for determining the required initial ratio of firepower assets of the warring parties under the given constraints (1):

$$K(0)_{необх} = \sqrt[4]{K^4(T)_{необх}(1-R(T)_{доп})^4 + \frac{\beta}{\alpha}(1-(1-R(T)_{доп})^4)}. \quad (10)$$

The second stage allows to calculate the projected duration of the operation (hostilities) under the same constraints (1).

Taking into account equations (3), (4), let us find the value of the integration constant from equation (8):

$$C = K^4(T)B^4(0)(1-R(T))^4 - \frac{\beta}{\alpha}B^4(0)(1-R(T))^4. \quad (11)$$

From (6), taking into account (11), one obtains:

$$A^2(t) = \sqrt{\frac{\beta}{\alpha}}\sqrt{B^4(t) + \frac{\alpha}{\beta}K^4(T)B^4(0)(1-R(T))^4 - B^4(0)(1-R(T))^4}. \quad (12)$$

For convenience, let's introduce a designation:

$$D = \frac{\alpha}{\beta}K^4(T)B^4(0)(1-R(T))^4 - B^4(0)(1-R(T))^4. \quad (13)$$

Substitute (12), taking into account (13), into the second equation of system (2). Then the result is:

$$\frac{dB(t)}{dt} = -\frac{\sqrt{\alpha\beta}}{B(t)}\sqrt{B^4(t)+D}, \quad (14)$$

from there

$$dt = -\frac{B(t)dB(t)}{\sqrt{\alpha\beta}\sqrt{B^4(t)+D}}. \quad (15)$$

Integrating expression (15), one obtains the following result:

$$t = -\frac{1}{2\sqrt{\alpha\beta}}\ln\left|B^2(t) + \sqrt{B^4(t)+D}\right| + C_1. \quad (16)$$

In function (16), the integration constant C_1 is determined from the initial conditions ($t=0$):

$$C_1 = \frac{1}{2\sqrt{\alpha\beta}}\ln\left|B^2(0) + \sqrt{B^4(0)+D}\right|. \quad (17)$$

Then for t, the formula (16) is written as follows:

$$t = \frac{1}{2\sqrt{\alpha\beta}} \ln \left| \frac{B^2(0) + \sqrt{B^4(0) + D}}{B^2(t) + \sqrt{B^4(t) + D}} \right|. \quad (18)$$

Taking into consideration equation (3),the required initial number of firepower assets $B(0)_{необх}$ for party *B* given a known initial value $A(0)$ for party *A* and the calculated value of $K(0)_{необх}$ is calculated using the following formula:

$$B(0)_{необх} = \frac{A(0)}{K(0)_{необх}}. \quad (19)$$

At the $t = T$ point in time equation (4) can be used to obtain the estimated number of firepower assets for party *B*:

$$B(T)_{розр} = B(0)_{необх}(1 - R(T)_{доп}). \quad (20)$$

Then, taking into account equation (18), the obtained values for $B(0)_{необх}$ and $B(T)_{розр}$, one can obtain the projected duration of the operation (hostilities) under the given constraints (1):

$$T = \frac{1}{2\sqrt{\alpha\beta}} \ln \left| \frac{B^2(0)_{необх} + \sqrt{B^4(0)_{необх} + D^*}}{B^2(T)_{розр} + \sqrt{B^4(T)_{розр} + D^*}} \right|. \quad (21)$$

where $D^* = \frac{\alpha}{\beta} K^4(T)_{необх} B^4(0)_{необх}(1 - R(T)_{доп})^4 - B^4(0)_{необх}(1 - R(T)_{доп})^4$.

It is important to highlight that the entire algorithm for critical system analysis (2) can be extrapolated to other instances of armed conflict, including small tactical-level operations.

Findings

Thus, the derivation of formulas (10) and (21) achieves the goal of the article.

To confirm the accuracy of the obtained equations (10) and (21), let us compare the results of their calculation with the results of the numerical integration of system (2).

Let's select the following set of initial data for the calculation. Known initial number of firepower assets *A(0)* = 2270 *standard units* for party *A* at the intensity of the actual destructive effect $\alpha = 0{,}01$ *умов.од./доба* and $\beta = 0{,}03$ *умов.од./доба*, for the $t = T$ point in time, the required ratio of the number of firepower assets of the parties is specified $K(T)_{необх} = 1$ as well as the acceptable magnitude of relative losses $R(T)_{доп} = 0{,}33$ for party *B*.

For a hypothetical operation (hostilities) between the warring parties *A* and *B*, one has to calculate:

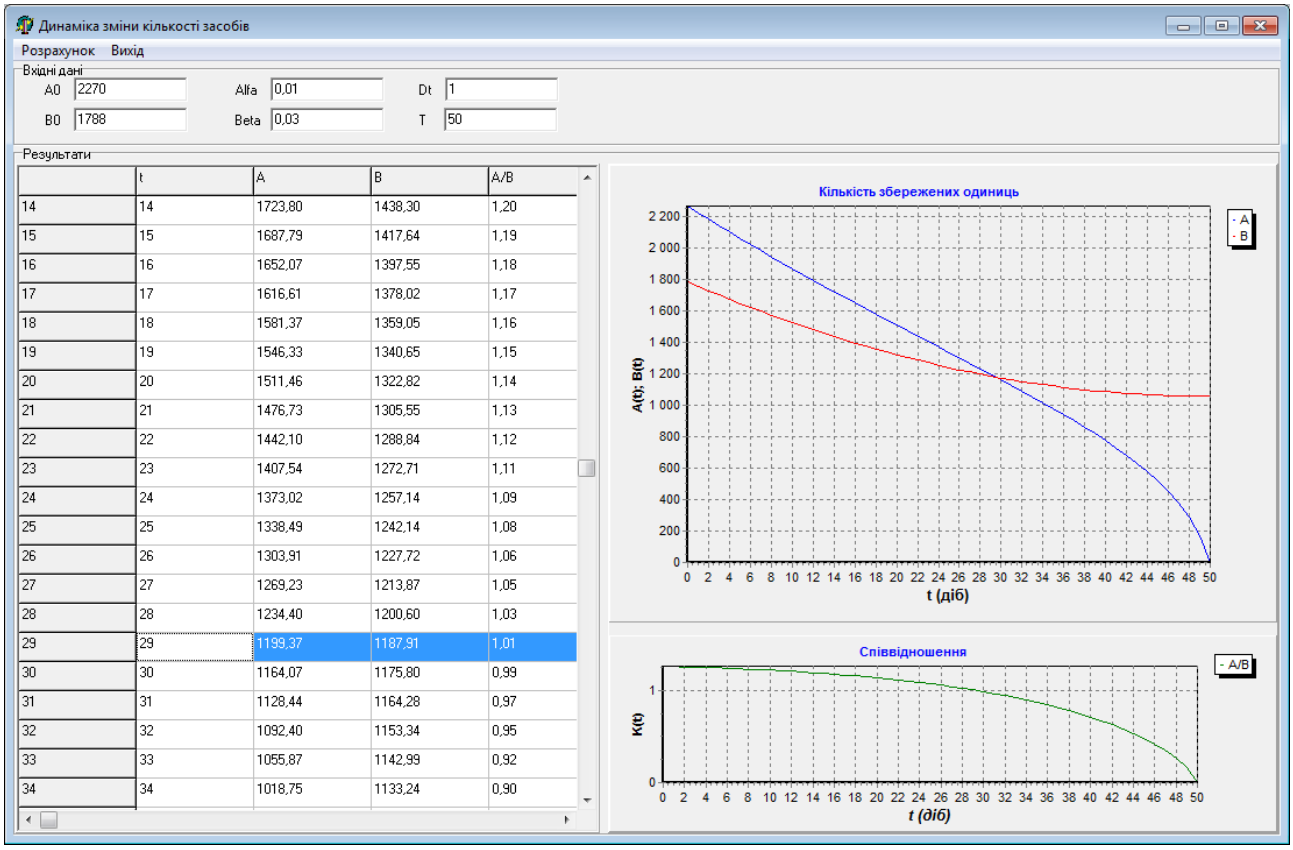
first, such an initial ratio of the number of firepower assets that will eventually yield the predetermined ratio $K(T)_{необх} = 1$, while meeting the condition of acceptable relative losses $R(T)_{доп} = 0{,}33$ for party *B;*

second, the projected duration of the operation (hostilities) T under the given constraints.

By plugging into equation (10) the values for $K(T)_{необх}$ and $R(T)_{доп}$, one gets the required

initial ratio of firepower assets of the warring parties $K(0)_{необх} \approx 1{,}3$. Using the obtained value $K(0)_{необх}$ in (19) to calculate the value of $B(0)_{необх} = 1788$ *умов.од.* that satisfies the given constraints (1). The projected duration of the operation (hostilities) is calculated using the formula (21) and amounts to $T \approx 29$ *діб*. For this purpose, the following values were used (21): $B(T)_{розр} = 1198$ *умов.од.*, calculations based on the formula (20), taking into account $B(0)_{необх} = 1788$ *умов.од.*, $R(T)_{доп} = 0{,}33$.

After numerical integration of the system (2) at *A(0)* = 2270 *standard units, B(0)* = 1788 *standard units* and the above given α, β (Figure. 1) for the duration of operation (hostilities) $T \approx 29$ *діб*, one gets $K(T)_{необх} \approx 1$ and $R(T)_{доп} \approx 0{,}33$. This result coincides with the values of the abovementioned analytical calculations, which indicates the validity of the estimates obtained using equations (10) and (21), which are the basis of the proposed approach.



	t	A	B	A/B
14	14	1723,80	1438,30	1,20
15	15	1687,79	1417,64	1,19
16	16	1652,07	1397,55	1,18
17	17	1616,61	1378,02	1,17
18	18	1581,37	1359,05	1,16
19	19	1546,33	1340,65	1,15
20	20	1511,46	1322,82	1,14
21	21	1476,73	1305,55	1,13
22	22	1442,10	1288,84	1,12
23	23	1407,54	1272,71	1,11
24	24	1373,02	1257,14	1,09
25	25	1338,49	1242,14	1,08
26	26	1303,91	1227,72	1,06
27	27	1269,23	1213,87	1,05
28	28	1234,40	1200,60	1,03
29	29	1199,37	1187,91	1,01
30	30	1164,07	1175,80	0,99
31	31	1128,44	1164,28	0,97
32	32	1092,40	1153,34	0,95
33	33	1055,87	1142,99	0,92
34	34	1018,75	1133,24	0,90

Figure 1. Numerical integration results

When performing calculations based on the abovementioned approach, one must bear in mind that describing the actions of the warring parties using systems of differential equations is to some extent a simplified interpretation of reality. Regardless, even in this case it is possible to calculate approximate values for different parameters of the operation (hostilities) that can used for planning purposes, particularly when justifying the required force composition and duration of the operation

Discussions

Dynamic averaging method is one of the tools used to study different dynamic processes taking place during a conflict (Venttsel, 1964), (Hrabchak, Mashkin, 2019), (Fursenko, 2020). It is centered

around the model of bilateral actions taken by parties, which is described by a system of differential equations. For systems of linear differential equations with a constant destructive effect, mathematical dependencies are obtained to determine the required initial ratio of firepower assets of the warring parties and the projected duration of combat operations under the predefined (acceptable) extend of losses (Venttsel, 1964), (Zahorko, 2020), (Romanchenko, 2020).

At the same time, the issue has been partially addressed for systems of nonlinear differential equations that more fully describe the nature of military conflict between the parties. In particular, researchers were able to obtain formulas describing dependencies only for the equations representing a prolonged operation (hostilities) with a slow decrease of available forces and assets over time (Kotliarov, 2020).

For systems that describe operations (hostilities) in a fast-paced environment, (Romanchenko, 2020) in his research emphasizes the importance of determining analytical dependencies that establish the relationship between the initial ratio of forces and assets of the warring party and the projected duration of hostilities between them that would produce an acceptable extent of losses for one of the parties while achieving the final predefined firepower ratio. However, no equations were derived describing these relationships. It is important to quickly calculate the aforementioned parameters that can be used in operation (combat) planning (in particular, when justifying the required force composition and duration of an operation (hostilities)).

Conclusions

The article further explores the field of critical analysis for Lanchester's nonlinear differential equations in the form of systems (2) used to describe fast-paced conflicts. In mathematical terms, such a conflict is described by a nonlinear model, where the right-hand sides of the differential equations are chosen to be a combination of the product of the current values of the combat power of the warring parties, which reflects the dynamic nature of the process to the fullest extent possible. Thus, the article proposes a double-pronged approach to critical analysis. The first aspect allows to establish a formula-driven relationship of the initial force ratio with predetermined criterion values. This relationship takes form of a biquadratic equation.

The second aspect concerns the critical assessment of the time required to achieve the ultimate objective of one of the parties in a conflict, where the original system is reduced to a single nonlinear first-order differential equation that can be solved for the time parameter using tabular integration.

For the first time, this approach is based on the analytical solution of the system of nonlinear differential equations with the dynamic averaging method, taking into account the possible changes in the intensity of the destructive effect delivered by forces and assets over the course of the hostilities, which is typical for modern armed conflicts.

Recommendations

Unlike the existing approaches, the proposed approach allows to perform calculations quickly without reducing their accuracy using the obtained formula-based dependencies. The practical applicability of this approach is demonstrated by the relevant computational example.

Further research should focus on developing recommendations on the application of this approach during operation planning and its integration into various sets of mathematical models.

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