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Comprehensive Mathematical Model of the Ballistic Trajectory of an Operational-Tactical Missile

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Abstract. Flat trajectories that do not exceed the Karman line and maneuvering in the terminal (descending) phase of the flight have become the main requirements for the characteristics of modern operational-tactical missiles. Fulfilling these requirements makes quasi-ballistic trajectories of operational-tactical missiles less predictable, reducing their detection range and the effectiveness of tracking and interception by modern air defense/ballistic missile defense systems. Such flight characteristics are currently possessed by Russian-made operational-tactical missiles such as the X-47M2 “Kinzhal”, 9M723 “Iskander-M,” and the North Korean KN-23 “Hwasong-11A”. A ballistic trajectory model with a controlled active phase has been developed to study the parameters of the quasi-ballistic trajectory of modern operational-tactical missiles. Its integration with the ballistic trajectory model based on Kepler's differential equations is carried out, which made it possible to calculate the operational-tactical missile parameters with the corrected active phase from the start to the endpoint. Based on the tactical and technical characteristics of the 9M723 “Iskander-M”, variants of its flight trajectory in the active phase were calculated using different pitch angle control programs: cubic (n = 3), quadratic (n = 2), and linear (n = 1) pitch angle versus time. The parameters of the pitch angle control programs for three variants of the trajectory apogee of the operational-tactical missile 9M723 “Iskander-M” are obtained: H_1 = 40 km, H_2 = 50 km, and H_3 = 60 km, and the corresponding graphs of parameters change (height-distance, speed-distance, speed-time) are presented. The developed model of the ballistic trajectory with a controlled active phase will further enhance adaptive algorithms for tracking quasi-ballistic missiles using defense acquisition radars and missile seekers. The purpose of this article is to develop and study a comprehensive mathematical model of the quasi-ballistic trajectory of modern operational-tactical missiles, which will combine a classic ballistic trajectory and a section with a corrected active flight. The main requirements for modeling the flight of an operational-tactical missile were that the missile's flight does not cross the Kármán line, taking into account various pitch angle control programs in the active section, as well as

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determining the influence of these programs on the main parameters of the trajectory (apogee, range, speed, and time characteristics). The results obtained will make it possible to further improve the accuracy of modeling and create the conditions for improving algorithms for radar detection, tracking, and interception of quasi-ballistic missiles by modern air defense/missile defense systems. The materials of the article can be used by scientists, engineers, and specialists in the field of radar, as well as by specialists in the field of air and missile defense. The results obtained during modeling can be useful for the following purposes: analysis of hypersonic aircraft as targets for air defense/missile defense systems; development of a mathematical model of the trajectory of a single hypersonic quasi-ballistic missile for air defense/missile defense systems trajectory processing algorithms; calculation of the trajectory parameters of modern operational-tactical ballistic missiles; methodology development for determining a set of characteristics of operational-tactical ballistic missiles for calculating the parameters of hypersonic quasi-ballistic flight trajectories.

Keywords: active phase of trajectory, control program, Euler's methods, Kapler's equations, quasi-ballistic trajectory.

Introduction

During the Russian-Ukrainian war, the Kh-47M2 “Kinzhal,” 9M723 “Iskander-M,” and KN-23 tactical missiles were widely used [1, 2]. These operational-tactical missiles belong to the class of hypersonic quasi-ballistic missiles – missiles that fly along flat trajectories and/or most of whose trajectory is ballistic with possible maneuvering or unexpected changes in direction and range, which makes them less predictable and difficult to intercept by modern air defense/missile defense systems [3]. The development and improvement of algorithms for tracking this class of operational-tactical missiles and guiding anti-aircraft guided missiles/anti-missiles to them requires an adequate model of a quasi-ballistic trajectory.

Several works [4–17] have been devoted to studying the parameters of ballistic missile trajectories of various classes. Works [4–6] present the structure of a mathematical model for operational-tactical missiles, focusing on the optimal flight range of a missile with a given payload. An approach is proposed for forming operational-tactical missiles' pitch angle control programs for different phases of the trajectory in the form of polynomials for vertical or inclined launch.

Articles [7, 8] are devoted to the study of the parameters of the flight trajectories of operational-tactical missiles of the Iskander-M, Fateh-110, and Zolfaghar types using a ballistic trajectory model based on Kepler's differential equations. An algorithm for determining the ballistic trajectory model is presented, which involves calculating the initial velocity and coordinates of the operational-tactical missiles at the end of the active phase of trajectory. Based on the tactical and technical characteristics of the above-mentioned missiles, a family of optimal and flat ballistic trajectories for different types of warheads has been constructed, and graphs of velocity and altitude changes during flight are presented.

In work [9], an analytical model of ballistic missile motion is proposed for simplified conditions, including a flat, non-rotating Earth surface. This expression enables the calculation of accurate mathematical values for trajectory parameters without the need for numerical integration of differential equations. A simplified exponential model of the atmosphere is used, but the drag coefficient of the rocket is expressed as a function of velocity rather than considered a constant. The use of the proposed analytical equations enables the direct calculation of the missile's speed at any point along its ballistic trajectory, as well as the maximum axial acceleration and the altitude at which it occurs.

Article [10] examines various models of maneuvering missile motion. The authors focus on creating motion models that can reproduce a wide range of maneuvers, such as parabolic and spiral motion, as well as intelligent evasive maneuvers.

Article [11] describes various ballistic missile motion models used in trajectory tracking algorithms:

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1) simplified models of ballistic trajectories based on classical equations of flight under the influence of gravity without considering atmospheric drag;

2) models that take aerodynamic drag into account, in which aerodynamic drag is considered in addition to the effects of gravity;

3) high-precision models that consider the full range of rocket movements: linear and angular acceleration, roll, pitch, and yaw angles, the effect of rotation and instability in flight.

The authors of [12] investigate a pitch angle control program for a two-stage rocket based on the condition of ensuring maximum flight range while limiting the maximum speed at the moment of stage separation. The permissible load on the rocket design at the point of completing the active phase of trajectory is taken into account. Two-parameter and three-parameter pitch angle control programs for multistage rockets are considered.

Works [13–16] consider the main approaches to optimizing the design of reentry vehicles for atmospheric and extra-atmospheric phases of trajectory, namely:

1) dependence of the trajectory shape on the choice of the pitch angle control program on the active phase of trajectory;

2) condition for vertical launch of the missile, as the most effective in terms of energy consumption and the impact of the “wave crisis”;

3) requirements for limiting the angular velocity of the ballistic missile by pitch angle;

4) limitation of the maximum flight range at a given guidance accuracy.

All the above-mentioned works consider separate phases of the quasi-ballistic trajectory: the controlled active phase [4–6], the mid-course phase [7, 8, 9, 11], and the controlled aerodynamic phase [10, 17]. The requirements presented in works [12–16] are relevant for intercontinental ballistic missiles or operational-tactical missiles that lack maneuverability in the terminal phase of their trajectory. None of the listed works provides a complete picture of the trajectory parameters of hypersonic quasi-ballistic missiles in different phases.

The purpose of this article is to develop a comprehensive model of a ballistic trajectory with a corrected active phase and to study the influence of various pitch angle control programs on the trajectory parameters of an operational-tactical missile.

Methodological Framework

To describe the flight of a rocket in the passive phase along a ballistic trajectory, we will use the algorithm for determining the ballistic flight trajectory model of an operational-tactical missile, proposed in [7]. We will describe the motion of the rocket's center of mass relative to the Earth, based on Kepler's equations in a rectangular coordinate system:

$$\dot{V}_{X_E}=\left[-\frac{GM_E}{r_E^2}-\frac{3B}{r_E^4}(5sin^2\varphi\text{-}1)+\omega_E^2 r_E\right]\frac{X_E}{r_E}-\frac{\gamma\rho}{2}V\dot{X}_E+2\omega_E\dot{Z}_E; \quad (1)$$

$$\dot{V}_{Y_E}=\left[-\frac{GM_E}{r_E^2}-\frac{3B}{r_E^4}(5sin^2\varphi\text{-}3)\right]\frac{Y_E}{r_E}-\frac{\gamma\rho}{2}V\dot{Y}_E; \quad (2)$$

$$\dot{V}_{Z_E}=\left[-\frac{GM_E}{r_E^2}-\frac{3B}{r_E^4}(5sin^2\varphi\text{-}1)+\omega_E^2 r_E\right]\frac{Z_E}{r_E}-\frac{\gamma\rho}{2}V\dot{Z}_E+2\omega_E\dot{Z}_E; \quad (3)$$

where X_E, Y_E, Z_E – coordinates operational-tactical missiles;

V_{X_E}, V_{Y_E}, V_{Z_E} – linear velocities along the axes OX_E, OY_E, OZ_E geocentric coordinate system;

G=$66.743 \cdot 10^{-12}$ $\frac{m^3}{kg \cdot s^2}$ – Newtonian gravitational constant;

M_E=$5.9722 \cdot 10^{24}$ kg – mass of the Earth;

r_E=$\sqrt{X_E^2+Y_E^2+Z_E^2}$ – distance between the centers of mass of the Earth and operational-tactical missiles;

$3B$=$26.245 \cdot 10^{24}$ $\frac{m^5}{s^2}$;

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φ – geocentric latitude of position operational-tactical missiles ($sin\varphi=Z_E/r_E$);

$\omega_E=\frac{2\pi}{86164}\ \frac{rad}{s}$ – angular velocity of Earth's rotation;

$g=\frac{GM_E}{r_E^2}$ – acceleration due to gravity;

V – linear velocity operational-tactical missiles;

$R_E=6371000\ m$ – radius of the Earth;

ρ– atmospheric density at altitude $H_E=r_E-R_E$ (for determining ρ at a height $H_E\ \leq 84\ km$ used built-in Matlab function atmoscoesa);

$\gamma=\frac{C_X S}{m}$ – ballistic coefficient;

C_X – aerodynamic drag coefficient, which depends most significantly on the shape of operational-tactical missile (ratio of the radius of curvature of the main part to amidships d), and for supersonic flight speeds, it is practically independent of the Mach number;

S – amidships square;

m – mass of operational-tactical missiles in the passive flight phase.

The generalized algorithm for determining the ballistic flight trajectory model of operational-tactical missiles includes the following stages [7]:

1) entering initial coordinate values for operational-tactical missiles;

2) determination of the matrix of guiding cosines between the axes of the starting and terrestrial coordinate systems;

3) determination of the velocity and coordinates of operational-tactical missiles at the end of the active phase of trajectory;

4) calculation of the components of the total velocity vector of operational-tactical missiles at the beginning of the passive phase of trajectory in the launch coordinate system;

5) recalculation of the initial values of the coordinates and components of the total velocity vector of operational-tactical missiles in the Earth coordinate system;

6) calculation

the distance between the centers of mass of the Earth and operational-tactical missiles;

altitudes of operational-tactical missile above the Earth's surface H_E;

atmospheric density ρ at a height H_E;

geocentric latitude of operational-tactical missiles.

1) solving differential equations (1) – (3) using Euler's method.

Results and Discussions

The development of a ballistic trajectory model with a corrected active phase of trajectory involves integrating the pitch angle control program for operational-tactical missiles into a generalized algorithm for determining the ballistic trajectory model. We will use the approach to forming pitch angle control programs for operational-tactical missiles in the form of a polynomial [4 – 6]:

$$\theta(t)=\sum_{j=0}^{n} A_j \cdot t^j, \qquad (4)$$

where $A_j, j=\overline{0,n}$ – coefficient determined depending on the selected control program;

n – a value that shows the degree of the polynomial.

On the active phase of trajectory, the following can be used as a pitch angle control program cubic ($n = 3$), quadratic ($n = 2$) i linear ($n = 1$) dependency of the pitch angle on time.

The main parameters on which the values of the polynomial coefficients depend are:

1) φ_{st} – starting angle operational-tactical missiles;

2) t_{st} – flight time operational-tactical missiles at the starting angle;

3) t_{AST} – flight time of operational-tactical missiles on the active phase of trajectory, equal to the engine operating time t_{EO};

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4) φ_{AST} – pitch angle of operational-tactical missile at the end of the active phase of trajectory.

Parameters 1–3 are determined by the technical characteristics of operational-tactical missiles, while parameter 4 is determined depending on the specified requirements for maximum altitude (apogee) and flight range.

The equations for calculating the coefficients of the polynomial (4) of different degrees are shown in Table 2.

To integrate the pitch angle control program for operational-tactical missiles into the generalized algorithm for determining the ballistic trajectory model, steps 3 and 4 of the algorithm must be modified [7].

Step 3 determines the speed V and coordinates operational-tactical missiles X_q, Y_q, Z_q on the active phase of trajectory by solving the differential equation [12]:

$$\dot{V}(t_i)= \frac{Jm_{\Pi_i}}{t_{EO}m_i} - \frac{C_X qS}{m_i} - g \cdot sin\theta(t_i), \quad (5)$$

where t_{EO} – engine operating time operational-tactical missiles;

$m_i=m_0-m_{\Pi_i}$ – current mass of operational-tactical missiles, calculated as the difference between the launch mass m_0 and fuel mass m_{Π_i}, spent at time i $t_i=\overline{0, t_{EO}}$;

$\theta(t_i)$ – pitch angle of operational-tactical missiles at time i.

The differential equation (5) is solved using Euler's method:

$$V(t_{i+1})=V(t_i)+\Delta t \cdot F_q,$$

where F_q – the right side of equation (5) at the value of the pitch angle $\theta(t_i)$;

Δt – integration step, which depends on the required accuracy of solving equation (5).

In step 4, the components of the total velocity vector of operational-tactical missiles at the end of the active phase of trajectory in the launch coordinate system are calculated:

$$\begin{aligned} V_{X_q}&=V_{AST} \cdot cos\theta_{AST} \cdot sinA; \\ V_{Y_q}&=V_{AST} \cdot sin\theta_{AST}; \\ V_{Z_q}&=-V_{AST} \cdot cos\theta_{AST} \cdot cosA; \end{aligned} \quad (6)$$

where V_{AST} та θ_{AST} – accordingly, the speed and pitch angle of operational-tactical missiles at the end of the active phase of trajectory;

A– azimuth of fire (angle between the meridian passing through the launch point and the projection of the flight path of operational-tactical missiles onto the Earth's surface).

For data input into modeling, we will utilize the tactical and technical characteristics of the 9M723 Iskander-M (Table 1) [7]. The pitch angle at the end of the active phase of trajectory is determined from the flight condition of operational-tactical missiles at maximum range, with the following apogee options: H_1 = 40 km, H_2 = 50 km, and H_3 = 60 km. The results of modeling the parameters of the ballistic trajectory with a corrected active phase of trajectory for different pitch angle control programs are shown in Fig. 1 and Table 2.

Table 1. Characteristics of the 9M723 Iskander-M missile

Characteristics	Missile weight, kg	Fuel weight, kg	Warhead weight, kg	Engine operating time, s	Specific impulse of engine thrust, km/s	Coefficient of drag	φ_{st}, deg	t_{st}, s
Value	3 320	2 988	480	25	2,75	0,25	90	15

Note: developed by the authors based on data from [7]

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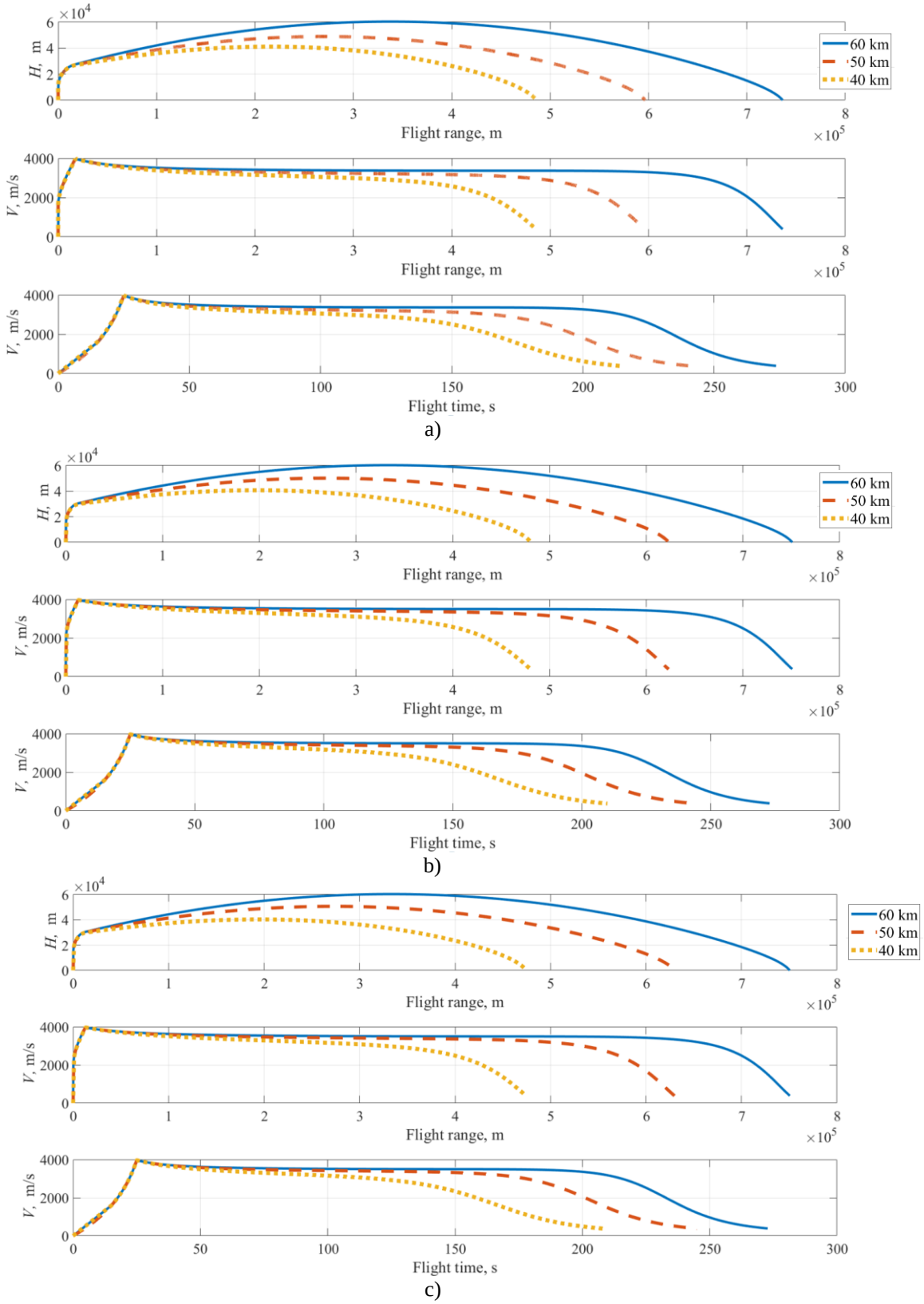


Figure 1. Ballistic trajectory parameters with the corrected active phase of trajectory for different pitch angle control programs:
a) $n=1$; b) $n=2$; c) $n=3$

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Table 2. Results of ballistic trajectory modeling with the corrected active phase of trajectory for different pitch angle control programs

Degree of a polynomial	$n=1$			$n=2$			$n=3$		
Coefficients of a polynomial	$A_0=\varphi_{st}-A_1\cdot t_{st}$ $A_1=\frac{\varphi_{st}-\varphi_{AST}}{t_{st}-t_{AST}}$			$A_0=\varphi_{st}-(A_1\cdot t_{st}+A_2\cdot t_{st}^2)$ $A_1=-2\cdot A_2\cdot t_{st}$ $A_2=\frac{\varphi_{st}-\varphi_{AST}}{(t_{st}^2-t_{AST}^2)-2\cdot t_{st}\cdot(t_{st}-t_{AST})}$			$A_0=\varphi_{st}-A_1\cdot t_{st}-A_2\cdot t_{st}^2-A_3\cdot t_{st}^3$ $A_1=-3\cdot A_3\cdot t_{st}^2-2\cdot A_2\cdot t_{st}$ $A_2=\frac{\varphi_{AST}-A_3\cdot t_{AST}^3-\varphi_{st}-A_3\cdot t_{st}^3}{(t_{st}-t_{AST})^2}-$ $-\frac{3\cdot A_3\cdot t_{st}^2\cdot(t_{st}-t_{AST})}{(t_{st}-t_{AST})^2}$ A_3 – is selected so that operational-tactical missiles reach the desired angle of attack at the end of the active phase of trajectory [3]		
Apogee, km	40	50	60	40	50	60	40	50	60
φ_{AST}, deg	19	21,8	23,8	14,8	18	19,8	14,8	18	19,8
Speed at the end of AST, km/s	3,50	3,49	3,49	3,55	3,56	3,56	3,55	3,56	3,56
Height at the end of the AST, km	22,6	23,1	23,6	25,2	25,7	25,9	25,1	25,6	25,9
Degree of a polynomial	$n=1$			$n=2$			$n=3$		
Apogee, km	40	50	60	40	50	60	40	50	60
Missile flight time, s	266	298	326	266	303	327	264	302	326
Maximum range, km	185	258	332	203	303	373	199	298	368

Conclusion and Discussions

The article presents a model of the ballistic trajectory of operational-tactical missiles with the corrected active phase. The calculation of the flight parameters of operational-tactical missiles in the active phase of trajectory was performed using various pitch angle control programs: cubic (n=3), quadratic (n=2), and linear (n=1) dependence of the pitch angle on time. A partial model of the active phase of trajectory of operational-tactical missiles was integrated into the ballistic trajectory model, enabling the calculation of its parameters from the launch point to the point of impact.

The parameters of the pitch angle control programs were obtained for three variants of the apogee of the trajectory of the 9M723 Iskander-M operational-tactical missiles: H_1 = 40 km, H_2 = 50 km, and H_3 = 60 km.

Using the developed comprehensive model of the ballistic trajectory of operational-tactical missiles with the corrected active phase, the trajectory parameters were calculated, and the corresponding dependencies of the range and flight time of operational-tactical missiles on speed and altitude were constructed.

Promising areas for further research include:

development of a model for the motion of operational-tactical missiles in the controlled aerodynamic flight phase;

development of models of maneuvering operational-tactical missiles in the controlled

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aerodynamic flight phase;

development of a comprehensive model of a quasi-ballistic trajectory.

Conflict of Interest

The authors declare that they have no competing interests related to the subject of the article.

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Анотація. Настильні траєкторії, що не виходять за межі лінії Кармана, та маневрування на кінцевій (термінальній) фазі польоту стали основними вимогами до характеристик сучасних оперативно-тактичних ракет. Виконання цих вимог робить квазібалістичні траєкторії оперативно-тактичних ракет менш передбачуваними, зменшуючи дальність їх виявлення та ефективність супроводження і перехоплення сучасними системами протиповітряної/протиракетної оборони. Такими льотними характеристиками на даний момент володіють оперативно-тактична ракета російського виробництва 9М723 "Іскандер-М" та північнокорейська KN-23 "Hwasong-11A", а також аеробалістична ракета Х-47М2 "Кинджал". Для дослідження параметрів квазібалістичної траєкторії сучасних оперативно-тактичних ракет розроблено модель балістичної траєкторії з коригованою активною ділянкою. Здійснено її інтеграцію з моделлю балістичної траєкторії на основі диференціальних рівнянь Кеплера, що дозволило розрахувати параметри оперативно-тактичної ракети з коригованою активною ділянкою траєкторії від точки старту до точки падіння. На основі тактико-технічних характеристик оперативно-тактичної ракети 9М723 "Іскандер-М" розраховано варіанти її траєкторії польоту на початковій фазі з використанням різних програм управління кутом тангажу: кубічної ($n = 3$), квадратичної ($n = 2$) та лінійної ($n = 1$) залежно від часу. Отримано параметри програм управління кутом тангажу для трьох варіантів апогею траєкторії оперативно-тактичної ракети 9М723 "Іскандер-М": $H_1 = 40$ км, $H_2 = 50$ км і $H_3 = 60$ км, а також представлено відповідні графіки зміни параметрів (висота-дальність, швидкість-дальність, швидкість-час). Розроблена модель балістичної траєкторії з корегованою активною ділянкою допоможе в подальшому вдосконалювати адаптивні алгоритми супроводження квазібалістичних ракет алгоритмами радіолокаційних станцій та головок самонаведення

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ракет. Метою даної статті є розроблення та дослідження комплексної математичної моделі квазібалістичної траєкторії сучасних оперативно-тактичних ракет, яка буде в собі поєднувати класичну балістичну траєкторію і ділянку з коригованим активним польотом. Основними вимогами до моделювання польоту оперативно-тактичної ракети були, такі як траєкторія польоту не перевищує висоти лінії Кармана, врахування різних програм управління кутом тангажу на активній ділянці, а також визначення впливу цих програм на основні параметри траєкторії (апогей, дальність, швидкість і часові характеристики). Отримані результати дозволять в подальшому підвищити точність моделювання та реалізують передумови для вдосконалення алгоритмів радіолокаційного виявлення, супроводження та перехоплення квазібалістичних ракет сучасними системами ППО/ПРО. Матеріали статті можуть бути корисними для науковців, інженерів і фахівців у сфері радіолокації, а також для спеціалістів у сфері протиповітряної і протиракетної оборони. Результати, отримані в ході моделювання, можуть бути корисними для наступних цілей: аналізу гіперзвукових літальних апаратів як цілей для систем протиповітряної/протиракетної оборони; розроблення математичної моделі траєкторії одиночної гіперзвукової квазібалістичної ракети для алгоритмів траєкторного оброблення засобів протиповітряної/протиракетної оборони; розрахунку параметрів траєкторій сучасних оперативно-тактичних балістичних ракет; розроблення методики визначення набору характеристик оперативно-тактичних балістичних ракет для розрахунку параметрів гіперзвукових квазібалістичних траєкторій польоту.

Ключові слова: активна ділянка траєкторії, програма управління, метод Ейлера, рівняння Каплера, квазібалістична траєкторія.